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NOMOGRAPHS FOR CALCULATING THE SECOND DERIVATIVES OF THE FORCE OF GRAVITY POTENTIAL OBTAINED BY OBSERVATIONS MADE WITH A TORSION BALANCE AT 5 AZIMUTHS

BY

W. AYVAZOGLOU
TRANSLATED FROM THE ORIGINAL RUSSIAN OF A. STEPANOFF

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NOMOGRAPHS FOR CALCULATING THE SECOND DERIVATIVES OF THE FORCE OF GRAVITY POTENTIAL OBTAINED BY OBSERVATIONS MADE WITH A TORSION BALANCE AT 5 AZIMUTHS[1]

Translated by W. Ayvazoglou[2] from the original Russian of A. Stepanoff

As we know, the calculation of the second derivatives of the potential of the force of gravity by observations made with a torsion balance at 5 azimuths can be carried out by the formulas:

$$\begin{aligned} 2\,V_{xy} &= K\,(u_2 + u_5) + l\,(u_3 + u_4), \\ V_{\Delta} &= m\,(u_2 - u_5) + n\,(u_3 - u_4), \\ V_{xz} &= o\,(u_2 - u_5) + p\,(u_3 - u_4), \\ V_{yz} &= q\,(u_2 + u_5) + r\,(u_3 + u_4), \end{aligned}$$

in which k, l, m, n, o, p, q, and r are the coefficients depending on the constants of the instrument.

$$u_i = m_i - m_o, \qquad i = 1, 2, 3, 4, 5,$$

and

$$m_o = \frac{m_1 + m_2 + m_3 + m_4 + m_5}{5},$$

in which m_i represents the readings of the positions of the beams for 0, 72, 144, 216, and 288° of the azimuths from the points shown by a fixed mirror. Although the use of these formulas does not offer great difficulties, the persons who make the calculations may consider the multiplication of the values $u_2 + u_5$, etc., by the corresponding coefficients, taking into consideration the signs of the coefficients and of the values $u_2 + u_5$, etc., very tiresome and slow.

This may be noted especially if a series of plates is to be worked out, as often happens in practice. In order to make this work easier the use of nomographs is proposed by which the value of the gradient can be found directly after obtaining the values of $u_2 + u_5$, etc.

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1 The Bureau of Mines will welcome reprinting of this paper, provided the following footnote acknowledgment is used: "Reprinted from U. S. Bureau of Mines Information Circular 6306."

2 Principal translator, U. S. Bureau of Mines.

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The construction of the offered nomographs is not complicated, but requires careful drawing.

An inconvenience results in the fact that four nomographs are necessary for each beam and that in case of the change of the filament new nomographs must be constructed because the values of the coefficients in the formulas usually change. But in most cases the constants of the two beams do not differ very much; therefore the beams may be considered to be equivalent if mean values of coefficients are taken. On the other hand, the same filament serves a sufficiently long time - during the whole period of field work and even longer. We must mention that usually the instruments are adjusted at 3 azimuths, but in certain cases it may be useful to adjust the instrument at 5 azimuths in order to judge whether the readings of the two beams agree with one another or not; this may serve for checking the instrument.

For our formulas the construction of simple nomographs consisting of three parallel graduated lines is very convenient. As we know, these nomographs can be constructed for a function of the kind: $z = Ax + By$. Nomographs of this type consist (fig. 1) of three parallel graduated lines on two of which the values of "x" and "y" are figured according to a certain scale, and on the third of which the value of z is drawn.

In order to find the value of z according to the data for "x" and "y", we determine the values of "x" and "y" on the corresponding graduated lines and connect these points with one another by a line (the line is not drawn actually on the plan, but a piece of transparent paper with a straight line drawn on it is put over the plan); after this we read the value of z on the third graduated line at the point of its intersection with the line.

The construction of these nomographs is based on the following formulas:

Let us have the function

$$z = Ax + By.$$

Let the distance between the graduated lines "x" and "y" (fig. 2) be equal to A, and the scales for "x" and "y" be equal to M_x and My. Let us construct the graduated lines, taking for their initial points their intersections with a certain horizontal line 0 - 0. The position of the graduated line z will in this case be determined by the relation

$$\delta = \frac{B\, M_x \cdot a}{A\, M_y + B\, M_x}$$

in which δ is the distance of the z-graduated line from the x-graduated line. The scale for the z-graduated line can be found from the formula

$$M_z = \frac{M_x\, M_y}{A\, M_y + B\, M_x}$$

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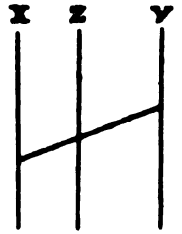

Figure 1

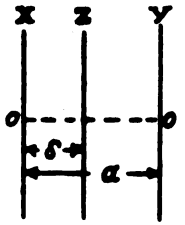

Figure 2

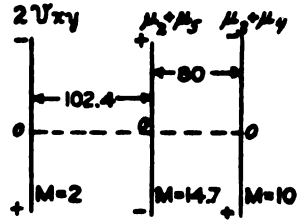

Figure 3

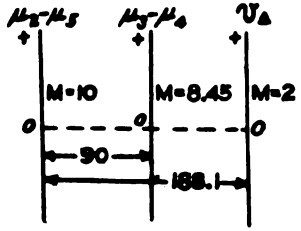

Figure 4

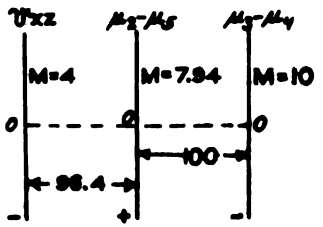

Figure 5

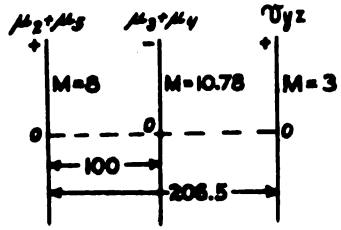

Figure 6

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After the construction of the z-graduated line we obtain our nomograph. For this construction the quantities M_x, M_y and a were taken as basic data. It is often more convenient to use the quantities M_x, M_z and a, as then the M_z-scale will be determined in advance. In this case we have:

$$\delta = a\left(1 - A\frac{M_z}{M_x}\right)$$

$$M_y = \frac{B\,M_x\,M_z}{M_x - A\,M_z}$$

By using M_y, M_z and a we find:

$$\delta = a\,B\,\frac{M_z}{M_y}$$

$$M_x = \frac{A\,M_y\,M_z}{M_y - B\,M_z}$$

If the quantity δ is negative, the z-graduated line will not be between the "x" and "y" graduated lines but at the side of them. If A and B are positive, then, in order to obtain the z-graduated line at the side of the two other graduated lines, we have to transform the equation $z = Ax + By$ into $z = (-A)(-x) + By$ or into $z = Ax + (-B)(-y)$.

It is important to construct the nomograph in such a way that the scales of all the graduated lines do not differ very much from one another. This is sometimes obtained by placing the z-graduated line outside of the "x" and "y" graduated lines.

In order to make the process of construction entirely clear, let us take an example:

For the first beam of torsion balance No. 563 (small Schweydar-Bamberg model, z-30) we obtain for certain filaments the following formulas:

$$\begin{aligned} 2\,V_{xy} &= -16.8\,(u_2 + u_5) - 6.42\,(u_3 + u_4), \\ V_{\Delta} &= -5.46\,(u_2 - u_5) + 8.84\,(u_3 - u_4), \\ V_{xz} &= -3.90\,(u_2 - u_5) + 2.41\,(u_3 - u_4), \\ V_{yz} &= -2.84\,(u_2 + u_5) - 7.42\,(u_3 + u_4). \end{aligned}$$

The signs of the formulas are taken under the assumption that the z-axis is directed vertically downward, the x-axis towards the south, the y-axis towards the west (or any other equivalent distribution of axes, for example: z-axis downward, x-axis towards the north, and y-axis towards the east) and that the increase of the readings of the position of the lever along the plate corresponds to the turning of the lever in the positive direction.

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Corresponding to the four formulas we have to construct four nomographs.

1. $2\,V_{xy} = -16.8\,(u_2 + u_5) - 6.42\,(u_3 + u_4)$.

We write the equation as follows:

$$-2\,V_{xy} = 16.8\,(u_2 + u_5) - 6.42\,\left[-(u_3 \quad u_4)\right].$$

By comparing it with the equation $z = Ax + (-B)(-y)$ we see that x^1 and y^1 are represented by $(u_2 + u_5)$ and $(u_3 + u_4)$ and that

$$A = 16.8,$$
$$B = -6.42.$$

Assuming the values for a, M_y and M_z to be equal to:

$$a = 80 \text{ mm.},$$
$$M_y = 10 \text{ mm.},$$
$$M_z = 2 \text{ mm.},$$

we find:

$$M_x = \frac{16.8 \cdot 2 \cdot 10}{10 + 6.42 \cdot 2} = 14.71 \text{ mm.},$$
$$\delta = \frac{-80 \cdot 6.42 \cdot 2}{10} = -102.4 \text{ mm.}$$

When the axes of the graduated lines $u_2 + u_5$ and $u_3 + u_4$ are drawn at a distance equal to 80 mm., we draw to the left from the axis of the $u_2 + u_5$-graduated line the axis of the z-graduated line at a distance equal to 102.4 mm. After the figuring of graduation on all the lines the nomograph is ready. In the same way we construct also the other nomographs.

2. $V_\Delta = -5.46\,(u_2 - u_5) + 8.84\,(u_3 - u_4)$

We have: $A = -5.46$; $B = +8.84$

Assuming: $a = 90$ mm. $M_x = 10$ mm. $M_z = 2$ mm., we find:

$\delta = 188.1$ mm.; $M_y = 8.45$ mm.,

and we obtain the following nomograph:

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3. $V_{xz} = -3.90\ (u_2 - u_5) - 2.41\ (u_3 - u_4)$,

or $-V_{xz} = 3.90\ (u_2 - u_5) + (-2.41)\ \left[-\ (u_3 - u_4)\right]$,

we find $A = +3.90$ $B = -2.41$;

Assuming: $a = 100$ mm.; $M_y = 10$ mm.; $M_z = 4$ mm.,

we have: $\delta = -96.4$ mm.; $M_x = 7.94$ mm.,

and we obtain the following nomograph:

4. $V_{yz} = -2.84\ (u_2 + u_5) - 7.42\ (u_3 - u_4)$,

or $-V_{yz} = -2.84\ \left[-(u_2 + u_5)\right] + 7.42\ (u_3 + u_4)$

Here: $A = -2.84$; $B = +7.42$;

Assuming: $a = 100$ mm.; $M_x = 8$ mm.; $M_z = 3$ mm.,

we find: $\delta = 206.5$ mm.; $M_y = 10.78$ mm.,

and the following nomograph is obtained:

By using the nomographs the graduated line may sometimes not be long enough in case of large gradients. Our quantities $u_2 - u_5$, etc., must then be divided by two or even four (the latter very seldom), apply them in the nomographs according to the obtained quotient, and multiply the result again by two or four.

The use of the inclosed working nomographs has shown us that under these conditions also the result necessary for the field work is sufficiently accurate.

Similar nomographs (figs. 7, 8, 9, 10) were constructed for instruments Nos. 562 and 563 and proved to be useful.

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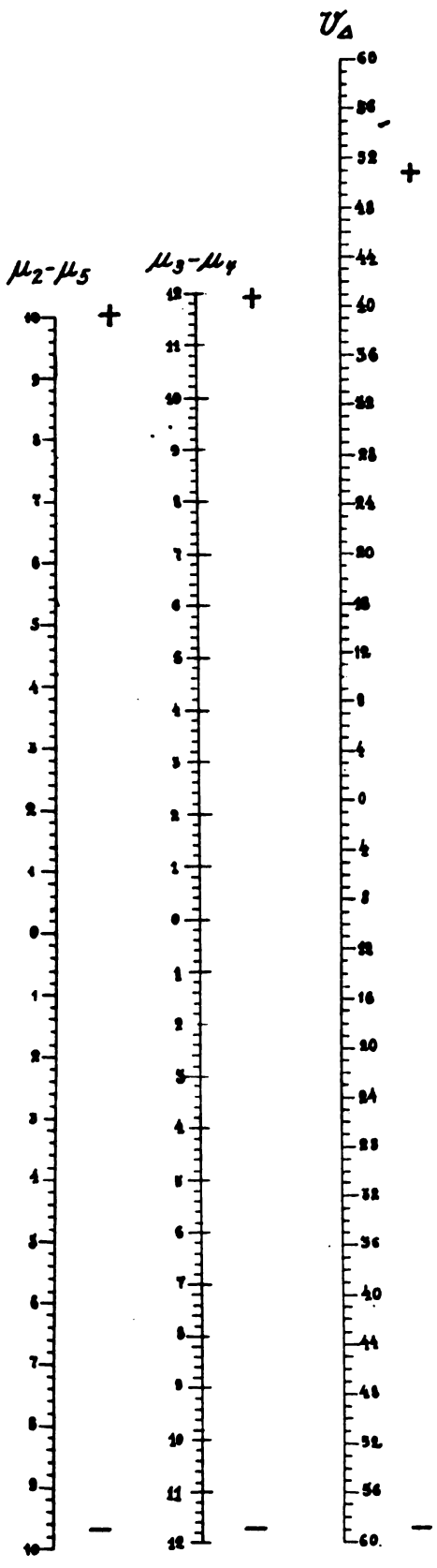

Figure 7.- Nomograph for ν_Δ

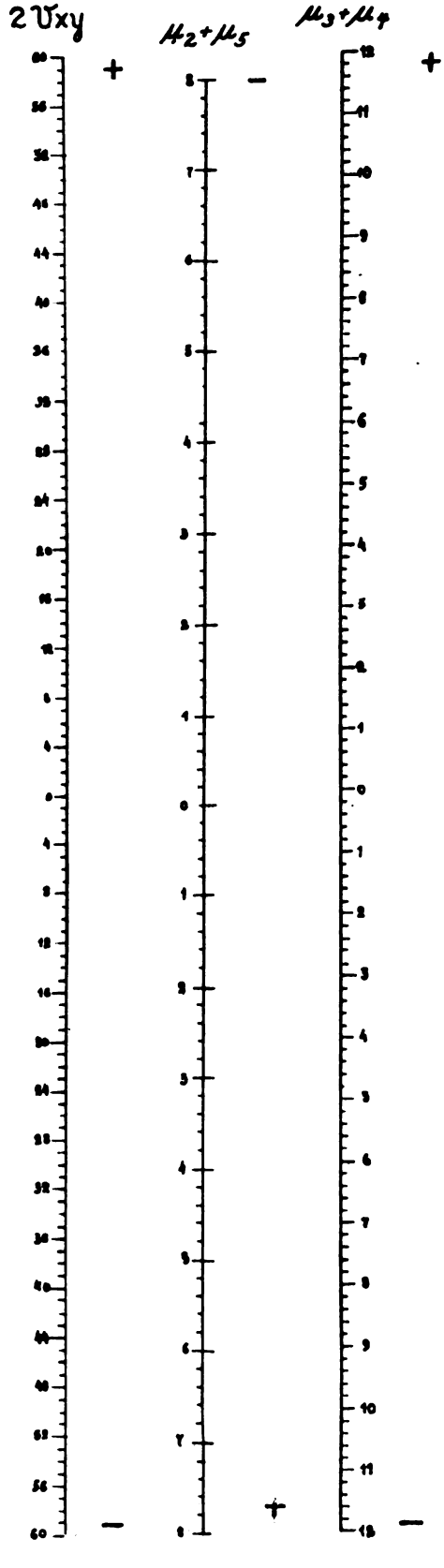

Figure 8.- Nomograph for $2\nu_{xy}$

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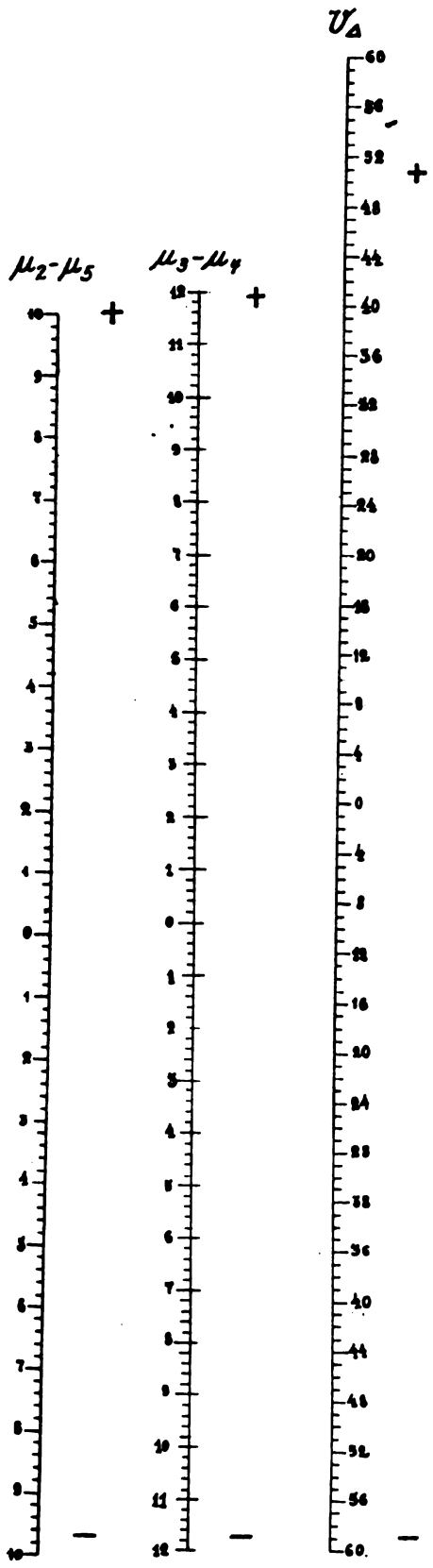

Figure 7.- Nomograph for ν_Δ

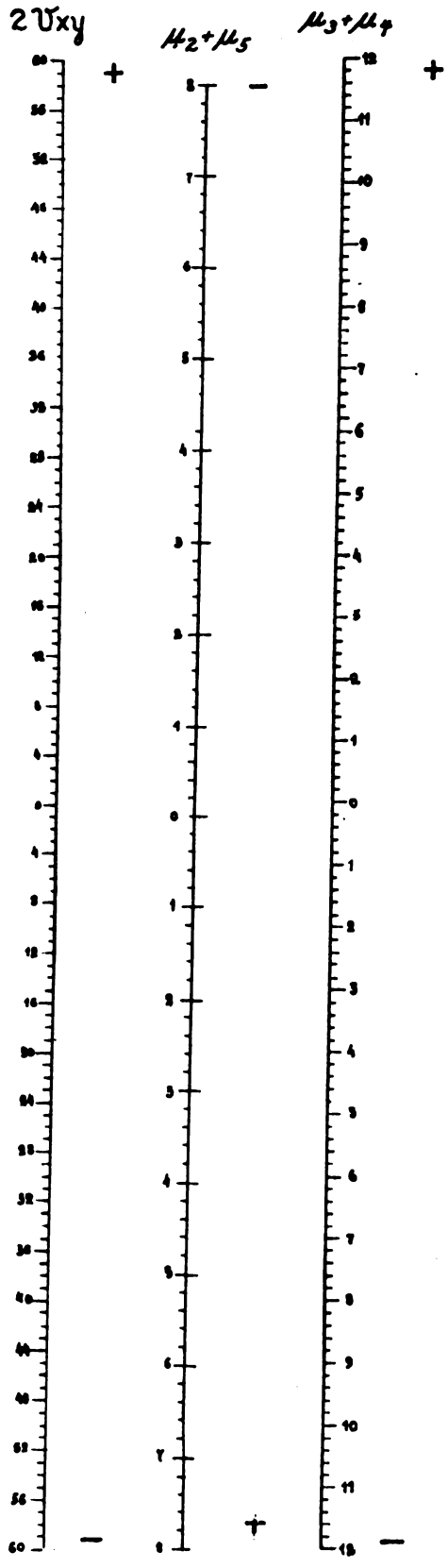

Figure 8.- Nomograph for $2\nu_{xy}$

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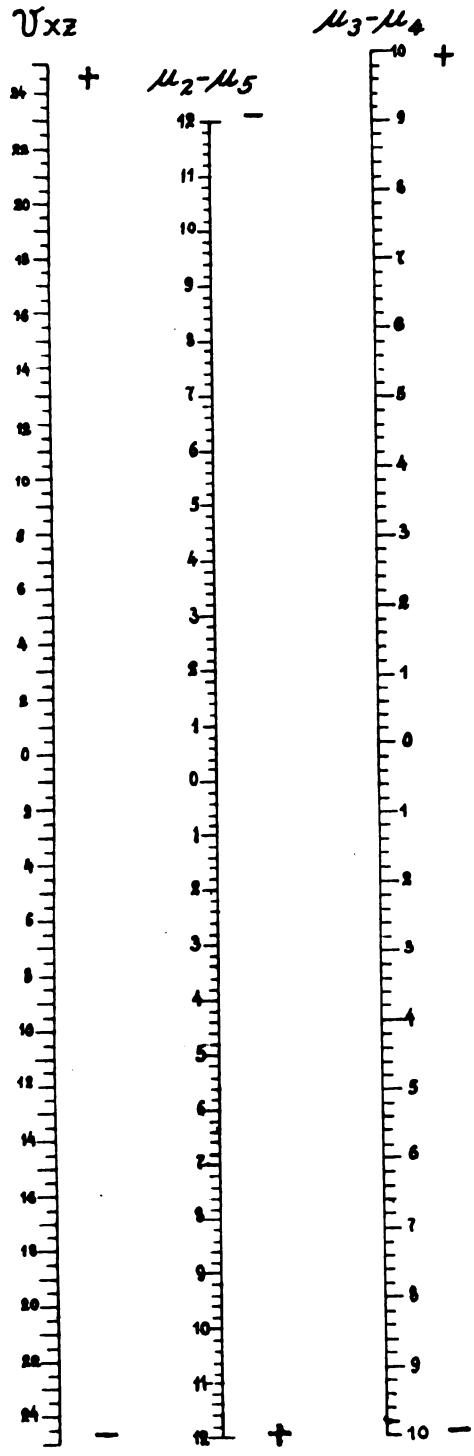

Figure 9.- Nomograph for ν_{xz}

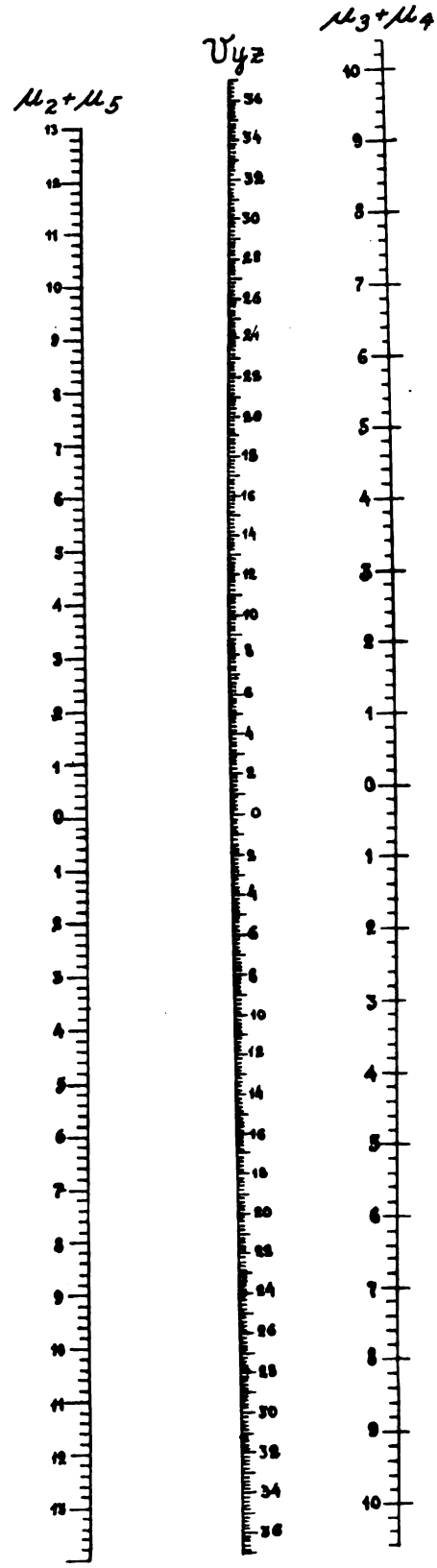

Figure 10.- Nomograph for ν_{yz}

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